Homework 3

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Instructions

- The maximum mark for this homework is 100.
- This homework consists of 4 short problem sets and each of these problem sets is worth 25% of the total homework mark (i.e., each problem set is worth 25 marks).
- Each problem set contains a number of bulleted questions. The number of bullets in each problem set is as follows:
  - Problem set 1 contains 13 bullets. Each bullet is therefore worth $\frac{25}{13} \approx 1.67$ marks.
  - Problem set 2 contains 15 bullets. Each bullet is therefore worth $\frac{25}{15} \approx 1.33$ marks.
  - Problem set 3 contains 3 bullets. Each bullet is therefore worth $\frac{25}{3} \approx 8.33$ marks.
  - Problem set 4 contains 4 bullets. Each bullet is therefore worth $\frac{25}{4} \approx 6.25$ marks.
- Please write your answers in the space provided below each bullet point or by annotating and labeling any R code, code output, or figures.
- If you require additional space, please use the back of the homework paper. If you use the back of the homework paper, please clearly indicate where your answer is located by writing a note in the space provided below the bullet.
- You may print this document and complete it by writing with pen or pencil. If you do this, you must scan your document to produce a pdf copy of your work. Submit the resulting pdf through iLearn.
- You may also use a tablet or touchscreen computer etc if you prefer. Anything is fine provided that it enables you create a pdf copy of your work that you can submit through iLearn.

Q1

Consider the following data from some experiment:

```
##             x        y
##         <num>    <num>
##   1: 37.92934 56.85227
##   2: 52.77429 58.96769
##   3: 60.84441 53.85514
##   4: 26.54302 59.00734
##   5: 54.29125 55.11681
##  ---
## 196: 55.00695 59.89394
## 197: 56.20210 56.57100
## 198: 40.34097 57.38833
## 199: 51.62655 52.14643
## 200: 29.21762 42.83511
```

- The `x` column contains dependent variable observations from an experiment condition named `x`.
- The `y` column contains dependent variable observations from an experiment condition named `y`.
- Suppose that we know that every observation from `x` or `y` was acquired from a different participant. Next, consider the following NHST result:

```
##
##  Two Sample t-test
##
## data:  x and y
## t = -3.2648, df = 398, p-value = 0.0005951
## alternative hypothesis: true difference in means is less than 0
## 95 percent confidence interval:
##       -Inf -1.638535
## sample estimates:
## mean of x mean of y
##  49.42241  52.73249
```

- State how each random variable that generates raw data in this experiment is distributed under the assumption that H_0 is true. Include numerical values for population parameters wherever possible.

- Draw each distribution from the previous bullet. Wherever possible, include labels that indicate population means.

- State all Null and Alternative hypotheses for this test.

- Write equations that state exactly how you will estimate the parameters used in all Null hypotheses stated in the previous bullet.

- For each null hypothesis included in this test, state how the random variable that generates the observed test statistic is defined under the assumption that H_0 is true.

- For each null hypothesis included in this test, what is the observed value of the test statistic? Plug the relevant numbers into the equations from the previous bullet. You do not have to evaluate the resulting expression exactly, but write down what you think it evaluates to approximately (this is so you can add it to a drawing in a later bullet).

- For each null hypothesis included in this test, state how the random variable that generates the test statistic is distributed under the assumption that H_0 is true. Include numerical values for population parameters wherever possible.

- For each null hypothesis included in this test, draw the distribution from the previous bullet. Add labels to this drawing that indicate upper and lower bounds, its population mean, and the observed value of the test statistic.

- For each null hypothesis included in this test, illustrate the p-value of the test by redrawing the appropriate distribution from previous bullets and shading the region that it corresponds to. Here, you do not need to calculate an exact numerical value.

- For each null hypothesis included in this test, illustrate the critical value(s) of this test by redrawing the appropriate distribution from previous bullets and marking the relevant location on the x-axis of the plot. Here, you do not need to calculate an exact numerical value.

- For each null hypothesis included in this test, illustrate its 95% confidence interval by redrawing the appropriate distribution from previous bullets and adding annotations.

- Write `R` code (using your pen or pencil) that would perform this NHST. Please only use functions and methods that I have directly covered in lectures.

- Please write a few sentences reporting the results of this analysis for an academic journal. Please include all relevant inferential statistic details as well as descriptive statistics for the mean(s) and standard error / confidence interval for the mean(s).

Q2

Consider a classic Stroop experiment in which colour words (e.g., “red”, “green”, “blue”) are presented in either congruent (e.g., the word “red” displayed in the colour red) or incongruent colours (e.g., the word “red” in the colour blue). After some data wrangling, the data from such an experiment may look like the following:

```
## Index: <trial_type>
##     subject  trial_type stim_colour response_time
##       <int>      <char>      <char>         <num>
##  1:       1   congruent         red     0.9045329
##  2:       1 incongruent         red     0.8070570
##  3:       1   congruent       green     0.9018390
##  4:       1 incongruent       green     0.8871968
##  5:       1   congruent        blue     1.4390432
##  6:       1 incongruent        blue     1.0884888
##  7:       2   congruent         red     1.1500053
##  8:       2 incongruent         red     1.0767804
##  9:       2   congruent       green     0.8588648
## 10:       2 incongruent       green     0.9352725
## 11:       2   congruent        blue     1.0414206
## 12:       2 incongruent        blue     0.8433074
## 13:       3   congruent         red     1.1630283
## 14:       3 incongruent         red     0.9907252
## 15:       3   congruent       green     1.0634876
## 16:       3 incongruent       green     0.9229206
## 17:       3   congruent        blue     0.8580425
## 18:       3 incongruent        blue     1.2521518
## 19:       4   congruent         red     1.0781808
## 20:       4 incongruent         red     0.9451513
## 21:       4   congruent       green     1.2179101
## 22:       4 incongruent       green     0.6565762
## 23:       4   congruent        blue     0.6587796
## 24:       4 incongruent        blue     1.0311982
## 25:       5   congruent         red     1.0866171
## 26:       5 incongruent         red     0.9754966
## 27:       5   congruent       green     1.3052747
## 28:       5 incongruent       green     0.9538796
## 29:       5   congruent        blue     1.2886127
## 30:       5 incongruent        blue     1.1344850
##     subject  trial_type stim_colour response_time
```

- Each row is an observation.
- The `subject` column indicates the subject from which each observation was obtained.
- The `trial_type` column indicates whether each observation was of a congruent or incongruent type.
- The `stim_colour` column indicates whether each observation was obtained from a red, green, or blue stimulus.
- The `response_time` column contains the mean response time observations observed for each subject and stimulus colour.

We also calculate the following descriptive statistics:

```r
dd <- d[, .(mean(response_time),
            sd(response_time)/sqrt(.N)),
        .(trial_type)]
```

```
dd
```

```
##     trial_type        V1         V2
##         <char>     <num>      <num>
## 1:   congruent 1.0677093 0.05307938
## 2: incongruent 0.9667125 0.03704191
```

To examine the classic Stroop effect, we need to formally compare response times on congruent trials to those on incongruent trials. After some further data wrangling – in which we collapse across stimulus colour and compute the difference in response times on congruent vs incongruent trials – we end up with the following:

```
##    subject diff_scores
##      <int>       <num>
## 1:       1 -0.15422420
## 2:       2 -0.06497676
## 3:       3  0.02707976
## 4:       4 -0.10731498
## 5:       5 -0.20554776
```

We finally perform an appropriate NHST to assess whether or not the Stroop effect is present in the data.

```
##
##  One Sample t-test
##
## data:  ddd[, diff_scores]
## t = -2.5447, df = 4, p-value = 0.06366
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.211192330  0.009198751
## sample estimates:
##  mean of x
## -0.1009968
```

- Please draw a pointrange plot with trial type (congruent vs incongruent) on the x-axis and the mean response time on the y-axis. Please use error bars to show SEM. Please label or annotate the plot with numerical values for means and SEMs.

- Please handwrite `R` code outlining how I calculated the `data.table` named `ddd` above.

- State how each random variable that generates raw data in this experiment is distributed under the assumption that H_0 is true. Include numerical values for population parameters wherever possible.

- Draw each distribution from the previous bullet. Wherever possible, include labels that indicate population means.

- State all Null and Alternative hypotheses for this test.

- Write equations that state exactly how you will estimate the parameters used in all Null hypotheses stated in the previous bullet.

- For each null hypothesis included in this test, state how the random variable that generates the observed test statistic is defined under the assumption that H_0 is true.

- For each null hypothesis included in this test, what is the observed value of the test statistic? Plug the relevant numbers into the equations from the previous bullet. You do not have to evaluate the resulting expression exactly, but write down what you think it evaluates to approximately (this is so you can add it to a drawing in a later bullet).

- For each null hypothesis included in this test, state how the random variable that generates the test statistic is distributed under the assumption that H_0 is true. Include numerical values for population parameters wherever possible.

- For each null hypothesis included in this test, draw the distribution from the previous bullet. Add labels to this drawing that indicate upper and lower bounds, its population mean, and the observed value of the test statistic.

- For each null hypothesis included in this test, illustrate the p-value of the test by redrawing the appropriate distribution from previous bullets and shading the region that it corresponds to. Here, you do not need to calculate an exact numerical value.

- For each null hypothesis included in this test, illustrate the critical value(s) of this test by redrawing the appropriate distribution from previous bullets and marking the relevant location on the x-axis of the plot. Here, you do not need to calculate an exact numerical value.

- For each null hypothesis included in this test, illustrate its 95% confidence interval by redrawing the appropriate distribution from previous bullets and adding annotations.

- Write `R` code (using your pen or pencil) that would perform this NHST. Please only use functions and methods that I have directly covered in lectures.

- Please write a few sentences reporting the results of this analysis for an academic journal. Please include all relevant inferential statistic details as well as descriptive statistics for the mean(s) and standard error / confidence interval for the mean(s).

Q3

Following on from Q2, suppose that we are interested in whether or not the colour of the stimulus has any influence over the magnitude of the Stroop effect.

- Please describe an appropriate statistical test to determine whether stimulus colour influences the magnitude of the Stroop effect. Please include in your description a statement about any assumptions this test makes and comment on why these assumptions are a good fit for this scenario.

- Please clearly and briefly state what you predict you would observe from this experiment and analysis. Please draw an appropriate plot or plots showing how data from this experiment would likely appear.

- Please write R code (using your pen or pencil) that would perform the analysis you have proposed.

Please only use functions and methods that I have directly covered in lectures. Please include a brief outline of how you will represent your data in a `data.table` object as well as how you will parse or pass that `data.table` object to an appropriate statistical test function (e.g., `t.test`, `ezANOVA`, `lm`).

Q4

Following on from Q2 and Q3, suppose that we are interested in whether or not the Stroop effect is influenced by either or both the colour of the stimulus and the font size of the stimulus. Our data might look like the following:

```
## Index: <trial_type>
##     subject  trial_type stim_colour font_size response_time
##       <int>      <char>      <char>    <char>         <num>
##  1:       1   congruent         red     small     1.1610359
##  2:       1 incongruent         red     small     2.4308636
##  3:       1   congruent       green     small     0.9211734
##  4:       1 incongruent       green     small     2.0758340
##  5:       1   congruent        blue     small     1.2727682
##  6:       1 incongruent        blue     small     2.8349568
##  7:       2   congruent         red     small     0.5958451
##  8:       2 incongruent         red     small     2.8090450
##  9:       2   congruent       green     small     1.4569421
## 10:       2 incongruent       green     small     2.3085714
## 11:       2   congruent        blue     small     1.3909700
## 12:       2 incongruent        blue     small     2.4953439
## 13:       3   congruent         red     small     1.0699962
## 14:       3 incongruent         red     small     2.5159344
## 15:       3   congruent       green     small     0.8721382
## 16:       3 incongruent       green     small     2.4585265
## 17:       3   congruent        blue     small     1.1011748
## 18:       3 incongruent        blue     small     2.5788904
## 19:       4   congruent         red     small     0.8677114
## 20:       4 incongruent         red     small     2.0365331
## 21:       4   congruent       green     small     1.3145086
## 22:       4 incongruent       green     small     2.8882048
## 23:       4   congruent        blue     small     1.1468891
## 24:       4 incongruent        blue     small     2.3836233
## 25:       5   congruent         red     large     1.0262482
## 26:       5 incongruent         red     large     0.9581202
## 27:       5   congruent       green     large     0.9298087
## 28:       5 incongruent       green     large     0.7103470
## 29:       5   congruent        blue     large     1.4629936
## 30:       5 incongruent        blue     large     1.2175262
## 31:       6   congruent         red     large     0.7596643
## 32:       6 incongruent         red     large     1.3502899
## 33:       6   congruent       green     large     1.2445121
## 34:       6 incongruent       green     large     0.9435352
## 35:       6   congruent        blue     large     1.2886795
## 36:       6 incongruent        blue     large     0.9644099
## 37:       7   congruent         red     large     0.9653482
## 38:       7 incongruent         red     large     1.0541279
## 39:       7   congruent       green     large     0.8907187
## 40:       7 incongruent       green     large     1.0847109
## 41:       7   congruent        blue     large     1.1889903
## 42:       7 incongruent        blue     large     0.9013587
## 43:       8   congruent         red     large     1.1719056
## 44:       8 incongruent         red     large     0.6935281
## 45:       8   congruent       green     large     0.9396813
## 46:       8 incongruent       green     large     1.2008979
```

```
## 47:       8   congruent        blue     large     0.9752208
## 48:       8 incongruent        blue     large     1.3680240
##     subject  trial_type stim_colour font_size response_time
```

We can investigate the design of this experiment with the following line:

```
d[, unique(subject), .(trial_type, stim_colour, font_size)]
```

```
##      trial_type stim_colour font_size    V1
##          <char>      <char>    <char> <int>
##  1:   congruent         red     small     1
##  2:   congruent         red     small     2
##  3:   congruent         red     small     3
##  4:   congruent         red     small     4
##  5: incongruent         red     small     1
##  6: incongruent         red     small     2
##  7: incongruent         red     small     3
##  8: incongruent         red     small     4
##  9:   congruent       green     small     1
## 10:   congruent       green     small     2
## 11:   congruent       green     small     3
## 12:   congruent       green     small     4
## 13: incongruent       green     small     1
## 14: incongruent       green     small     2
## 15: incongruent       green     small     3
## 16: incongruent       green     small     4
## 17:   congruent        blue     small     1
## 18:   congruent        blue     small     2
## 19:   congruent        blue     small     3
## 20:   congruent        blue     small     4
## 21: incongruent        blue     small     1
## 22: incongruent        blue     small     2
## 23: incongruent        blue     small     3
## 24: incongruent        blue     small     4
## 25:   congruent         red     large     5
## 26:   congruent         red     large     6
## 27:   congruent         red     large     7
## 28:   congruent         red     large     8
## 29: incongruent         red     large     5
## 30: incongruent         red     large     6
## 31: incongruent         red     large     7
## 32: incongruent         red     large     8
## 33:   congruent       green     large     5
## 34:   congruent       green     large     6
## 35:   congruent       green     large     7
## 36:   congruent       green     large     8
## 37: incongruent       green     large     5
## 38: incongruent       green     large     6
## 39: incongruent       green     large     7
## 40: incongruent       green     large     8
## 41:   congruent        blue     large     5
## 42:   congruent        blue     large     6
## 43:   congruent        blue     large     7
## 44:   congruent        blue     large     8
## 45: incongruent        blue     large     5
```

```
## 46: incongruent        blue     large     6
## 47: incongruent        blue     large     7
## 48: incongruent        blue     large     8
##      trial_type stim_colour font_size    V1
```

We wrangle our data to calculate the difference in response times on congruent vs incongruent trials for each subject and factor in our design. This is preparation for performing an appropriate NHST to assess whether or not the Stroop effect is influenced by either or both of our design factors.

```
##     subject stim_colour font_size response_time_difference
##      <fctr>      <fctr>    <fctr>                    <num>
##  1:       1         red     small               1.26982771
##  2:       1       green     small               1.15466062
##  3:       1        blue     small               1.56218852
##  4:       2         red     small               2.21319986
##  5:       2       green     small               0.85162926
##  6:       2        blue     small               1.10437390
##  7:       3         red     small               1.44593820
##  8:       3       green     small               1.58638830
##  9:       3        blue     small               1.47771555
## 10:       4         red     small               1.16882166
## 11:       4       green     small               1.57369620
## 12:       4        blue     small               1.23673418
## 13:       5         red     large              -0.06812804
## 14:       5       green     large              -0.21946172
## 15:       5        blue     large              -0.24546743
## 16:       6         red     large               0.59062561
## 17:       6       green     large              -0.30097691
## 18:       6        blue     large              -0.32426953
## 19:       7         red     large               0.08877967
## 20:       7       green     large               0.19399227
## 21:       7        blue     large              -0.28763155
## 22:       8         red     large              -0.47837751
## 23:       8       green     large               0.26121654
## 24:       8        blue     large               0.39280319
##     subject stim_colour font_size response_time_difference
```

Finally, we can perform an appropriate NHST to assess whether or not the Stroop effect is influenced by either or both the colour of the stimulus and the font size of the stimulus.

```
## $ANOVA
##                Effect DFn DFd           F            p p<.05        ges
## 2           font_size   1   6 471.3184696 6.236443e-07     * 0.83663980
## 3         stim_colour   2  12   0.3437584 7.158593e-01       0.05083510
## 4 font_size:stim_colour   2  12   0.1051354 9.010226e-01       0.01611616
##
## $`Mauchly's Test for Sphericity`
##                Effect         W         p p<.05
## 3         stim_colour 0.4316548 0.1224169
## 4 font_size:stim_colour 0.4316548 0.1224169
##
## $`Sphericity Corrections`
##                Effect       GGe     p[GG] p[GG]<.05       HFe     p[HF]
## 3         stim_colour 0.6376147 0.6267014           0.7330098 0.6543154
## 4 font_size:stim_colour 0.6376147 0.8128230           0.7330098 0.8426955
##   p[HF]<.05
```

```
## 3
## 4
```

- Please describe the statistical test used above to determine whether stimulus colour or font size influences the magnitude of the Stroop effect. Please include in your description a statement about any assumptions this test makes and comment on why or why not these assumptions are a good fit for this scenario.

- Please draw pointrange plots that could plausibly correspond to the results from this experiment. Please make one plot for each main effect and one for the interaction.

- Please write R code (using your pen or pencil) that would perform the analysis reported above. Please only use functions and methods that I have directly covered in lectures. Please include a brief outline of how you will represent your data in a `data.table` object as well as how you will parse or pass that `data.table` object to an appropriate statistical test function (e.g., `t.test`, `ezANOVA`, `lm`).

- Please write a few sentences reporting this result for an academic journal.